FPGA Implementation of Large Area Efficient and Low Power Geortzel Algorithm for Spectrum Analyzer

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Abstract—**Spectrum analysis is very essential requirement in instrumentation and communication signal interception .Spectrum analysis is normally carried out by online or offline FFT processing. But the FFT being highly mathematical intensive, is not suitable for low area and low power applications. Offline FFT processing can't give the real time spectrum estimation which is essential in communication signal interception. Online FFT computation takes very high resources, which makes the system costly and power hungry. The Goertzel algorithm is a digital signal processing (DSP) technique for identifying frequency components of a signal, published by Dr. Gerald Goertzel in 1958. While the general Fast Fourier transform (FFT) algorithm computes evenly across the bandwidth of the incoming signal, the Goertzel algorithm looks at specific, predetermined frequency. However the implementation of Goertzel algorithm for spectrum computation is not explored for FPGA implementation. The FPGA being capable of offering high frequency data paths in them become suitable for realizing high speed spectrum analysis algorithms.**

In this project Goertzel algorithm will be implemented as high Q band pass filter on FPGA reconfigurable architecture. VHDL will be used for code development. A digital frequency synthesizer produces frequency sweep which will drive the digital mixer. The digital mixer output is given to the Goerzel algorithm block. This algorithm output will be given to peak detection logic. The peak detector block output will be used for spectrum computation. The top level module integrates all these modules with appropriate clock and control circuitry. The results will be demonstrated by applying the deterministic signals such as SIN wave and also with random band limited signals. It will be aimed to achieve 32 steps in the band of operation for spectrum computation on Spartan 3E low cost FPGA. Modelsim tool will be used for simulation. Xilinx ISE will be used for synthesis and programming the FPGA. Xilinx Chipscope will be used on chip verification of results.

Keywords: **FFT, Goertzel algorithm, Spectrum Analysis, VHDL, XilinX ISE, Modelsim, Spartan 3E-FPGA board.**

I. Introduction

Spectrum Computation Techniques

A. Spectrum computation by FFT:

The spectrum is the basic measurement of an FFT analyzer. It is simply the complex FFT. Normally, the magnitude of the spectrum is displayed. The magnitude is the square root of the FFT times its complex conjugate. (Square root of the sum of the real (sine) part squared and the imaginary (cosine) part squared.) The magnitude is a real quantity and represents the total signal amplitude in each frequency bin, independent of phase. If there is phase information in the spectrum, i.e. the time record is triggered in phase with some component of the signal, then the real (cosine) or imaginary (sine) part or the phase may be displayed. The phase is simply the arctangent of the ratio of the imaginary and real parts of each frequency component. The phase is always relative to the start of the triggered time record.

Fourier's theorem states that any waveform in the time domain can be represented by the weighted sum of sines and cosines. The FFT spectrum analyzer samples the input signal, computes the magnitude of its sine and cosine components, and displays the spectrum of these measured frequency components. [2-5] For one thing, some measurements which are very hard in the time domain are very easy in the frequency domain. Consider the measurement of harmonic distortion. It's hard to quantify the distortion of a sine wave by looking at the signal on an oscilloscope. when the same signal is displayed on a spectrum analyzer, the harmonic frequencies and amplitudes are displayed with amazing clarity.

An FFT spectrum analyzer works in an entirely different way. The input signal is digitized at a high sampling rate, similar to a digitizing oscilloscope. Nyquist's theorem says that as long as the sampling rate is greater than twice the highest frequency component of the signal, the sampled data will accurately represent the input signal. To make sure that Nyquist's theorem is satisfied; the input signal passes through an analog filter which attenuates all frequency components above 156 kHz by 90 dB. This is the anti-aliasing filter. The resulting digital time record is then mathematically transformed into a frequency spectrum using an algorithm known as the Fast Fourier Transform, or FFT[10].

B. Real time sweep spectral extraction algorithm :

The system function block of a Real time sweep spectral extractor is illustrated in the below figure. The mixer combines the input test signal and reference signal from the local

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oscillator. The output modulated signals will have the sum and difference frequency signals[7].

The peak detector will detect the signal intensity, and send it to the X-Y scope. The sweep generator supports the x-axis signal, and pushes the local oscillator to generate the reference signal with the desired frequency. We adopt the Direct Digital Synthesis (DDS) technique to tune the reference signal .To meet the rather narrow band requirement, we need very high Q FIR band pass filter. It implies hundreds of tap orders.

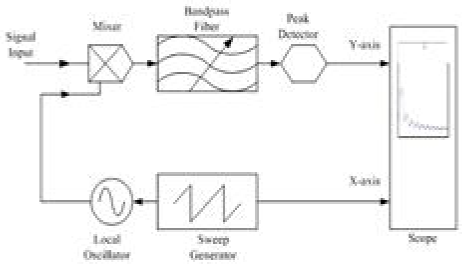

Fig 1.The system function block of a Real time sweep spectral extractor

C. Goertzel algorithm:

The Goertzel algorithm is a digital signal processing (DSP) technique for identifying frequency components of a signal, published by Dr. Gerald Goertzel in 1958. While the general Fast Fourier transform (FFT) algorithm computes evenly across the bandwidth of the incoming signal, the Goertzel algorithm looks at specific, predetermined frequencies. Some applications require only a few DFT frequencies. [1] One example is frequency-shift keying (FSK) demodulation, in which typically two frequencies are used to transmit binary data; another example is DTMF, or touch-tone telephone dialing, in which a detection circuit must constantly monitor the line for two simultaneous frequencies indicating that a telephone button is depressed. Goertzel's algorithm reduces the number of real-valued multiplications by almost a factor of two relative to direct computation via the DFT equation[9]. For a length of N, the Goertzel's series is

$$H_k(z) = \frac{1 - W_N^k z^{-1}}{1 - 2\cos(\frac{2\pi k}{N})z^{-1} + z^{-2}}$$

Where,

$$k = 0, 1, \ldots, N-1$$

The Goertzel algorithm computes a sequence, $s(n)$, given an input sequence, $x(n)$:

$$S(n) = x(n) + 2\cos(2\pi\omega)\, s(n-1) - s(n-2)$$

Where $s(-2) = s(-1) = 0$ and ω is some frequency of interest, in cycles per sample, which should be less than 1/2. This effectively implements a second-order one addition and one subtraction per input sample. For real inputs, these operations are real. The Z transform of this process is

$$\frac{S(z)}{X(z)} = \frac{1}{1 - 2\cos(2\pi\omega)z^{-1} + z^{-2}} = \frac{1}{(1 - e^{+2\pi i\omega}z^{-1})(1 - e^{-2\pi i\omega}z^{-1})}$$

Applying an additional, FIR, transform of the form[1]

$$\frac{Y(z)}{S(z)} = 1 - e^{-2\pi i\omega}z^{-1}$$

will give an overall transform of

$$\frac{S(z)}{X(z)}\frac{Y(z)}{S(z)} = \frac{Y(z)}{X(z)} = \frac{(1 - e^{-2\pi i\omega}z^{-1})}{(1 - e^{+2\pi i\omega}z^{-1})(1 - e^{-2\pi i\omega}z^{-1})} = \frac{1}{1 - e^{+2\pi i\omega}z^{-1}}$$

The time-domain equivalent of this overall transform is

$$y(n) = x(n) + e^{+2\pi i\omega}y(n-1) = \sum_{k=-\infty}^{n} x(k)e^{+2\pi i\omega(n-k)} = e^{+2\pi i\omega n}\sum_{k=-\infty}^{n} x(k)e^{-2\pi i\omega k}$$

Which becomes, assuming $x(k) = 0$ for all $k < 0$

$$y(n) = e^{+2\pi i\omega n}\sum_{k=0}^{n} x(k)e^{-2\pi i\omega k}$$

Or, the equation for the $(n + 1)$-sample DFT of x, evaluated for ω and multiplied by the scale factor $e^{+2\pi i\omega n}$. Note that applying the additional transform Y (z)/S (z) only requires the last two samples of the s sequence. Consequently, upon processing N samples $x(0)...x(N-1)$, the last two samples from the s sequence can be used to compute the value of a DFT bin, which corresponds to the chosen frequency ω as

$X(\omega) = y(N-1)e^{-2\pi i\omega(N-1)} = (s(N-1) - e^{-2\pi i\omega}s(N-2))e^{-2\pi i\omega(N-1)}$

For the special case often found when computing DFT bins, where $\omega N = k$ for some integer, k, this simplifies to

$X(\omega) = (s(N-1) - e^{-2\pi i\omega}s(N-2))e^{+2\pi i\omega} = e^{+2\pi i\omega}s(N-1) - s(N-2)$

In either case, the corresponding power can be computed using the same cosine term required to compute s as

$X(\omega)X'(\omega) = s(N-2)^2 + s(N-1)^2 - 2\cos(2\pi\omega)\, s(N-2)\, s(N-1)$

To compute a single DFT bin for a complex sequence of length N, this algorithm requires $2N$ multiplications and $4N$ additions/subtractions within the loop, as well as 4 multiplications and 4 additions/subtractions to compute $X(\omega)$, for a total of $2N+4$ multiplications and $4N+4$ additions/subtractions (for real sequences, the required operations are half that amount). In contrast, the Fast Fourier transform (FFT) requires $2\log_2 N$ multiplications and $3\log_2 N$ additions/subtractions per DFT bin, but must compute all N bins simultaneously .When the number of desired DFT bins, M, is small it is computationally advantageous to implement the Goertzel algorithm, rather than the FFT. Approximately, this occurs when

$$M < \frac{5}{6}\log_2 N$$

or if, for some reason, N is not an integral power of 2 while you stick to a simple FFT algorithm like the 2-radix Cooley-Tukey FFT algorithm, and zero-padding the samples out to an integral power of 2 would violate

$$M < \frac{5N_{\text{padded}}}{6N}\log_2(N_{\text{padded}})$$

Moreover, the Goertzel algorithm can be computed as samples come in, and the FFT algorithm may require a large table of N pre-computed sines and cosines to be efficient.

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II. Scope Of The Project

In this project we implement the Goertzel algorithm on FPGA using VHDL and implementing the spectrum estimation logic around Goertzel algorithm. As on today the spectrum computation is done with FFT based architectures. In case of online FFT computation the required resources are very high and power consumption is also high. In case of offline FFT computation real time applications can be built. The Proposed solution is that The Goertzel algorithm is used as band pass filter with a mixer stage preceding to it, for spectrum estimation. This will be low power and low area solution with respect to FFT.[1] The below figure shows the system function block of 1 real-time sweep spectral extractor with Goertzel algorithm.

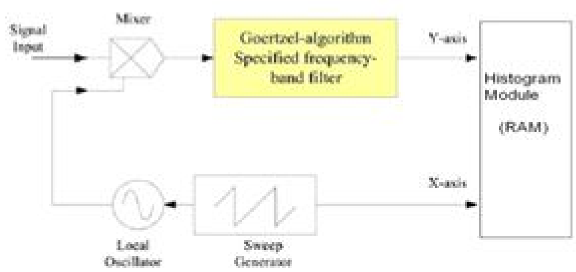

Fig 2. System function block of 1 real-time sweep spectral Extractor with Goertzel algorithm

The below figure is the corresponding second order recursive calculation flow. Here, it is apparently that Goertzel algorithm only needs two real multiplications and three real additions to pick up the amplitude of the specified frequency component.

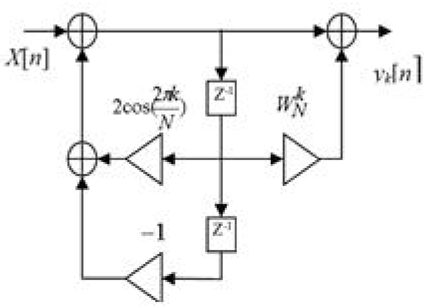

Fig 3 Calculation flow of the second order recursive algorithm

Conventionally for high frequency resolution we need narrow pass band of FIR filter and consume too much chip area. Without pass band narrowing, The Goertzel algorithm calculates and extracts the amplitude of the specified frequency component .We can achieve better efficiency without high chip area requirement.

III. Implementation Of Goertzel Algorithm

Goertzel algorithm:

For a length of N, the Goertzel's series is

$$H_k(z) = \frac{1 - W_N^k z^{-1}}{1 - 2\cos(\frac{2\pi k}{N})z^{-1} + z^{-2}}$$

Where,

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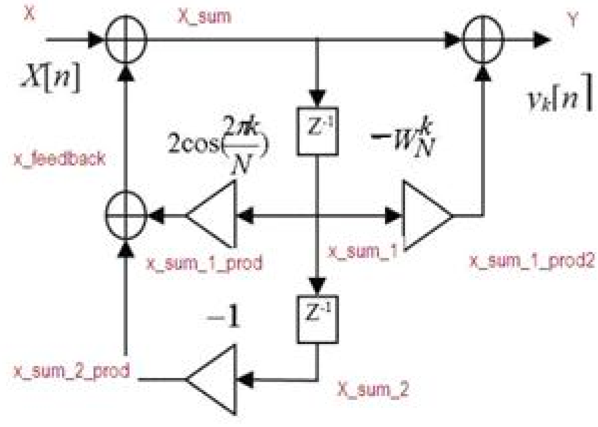
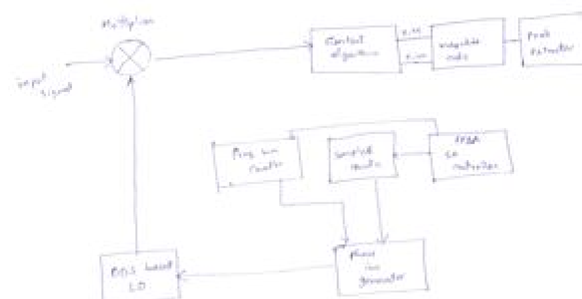


IV. Implementation Of Spectrum Analyzer

Top level design:

The detailed block diagram of the top level module is shown in the below fig.



In the above block diagram the DDS based LO generates the Numerically Controlled Oscillator (NCO) that will be one of the input signal to the mixer and the other input to mixer is the input test signal [11-14].The mixer will combine these two signals and generates a signal which is the difference between these two signals .The output signal of the mixer is at a frequency that the Goertzel algorithm is working and the Goertzel algorithm will identify the frequency components of the signal. The Goertzel algorithm will generate the frequency components of a signal in terms of the real and imaginary parts that will be applied to the input of the magnitude calculator followed by peak detector which determines the peak of the signal.

The FPGA Spectrum analyzer controller controls the all blocks like frequency bin counter and sample counter. The sample counter counts the samples, when ever this counter reaches the maximum value the frequency bin counter is incremented by one and the sample counter again starts counting until the entire spectrum is covered by the frequency bin counter. The Phase Increment generator generates the corresponding Phase increments for the DDS Core.

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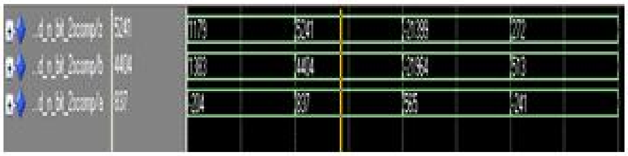
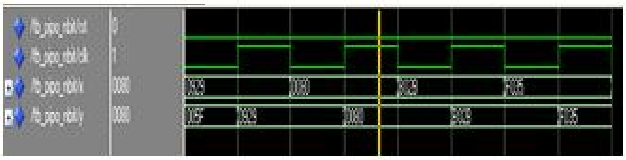
V. Simulation Results

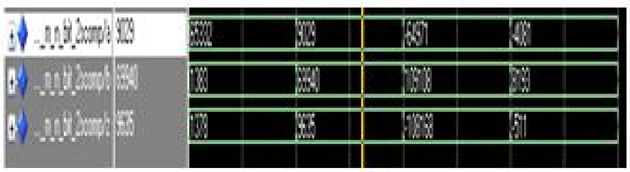
Adder:



PIPO Shift Register:



Multiplier:

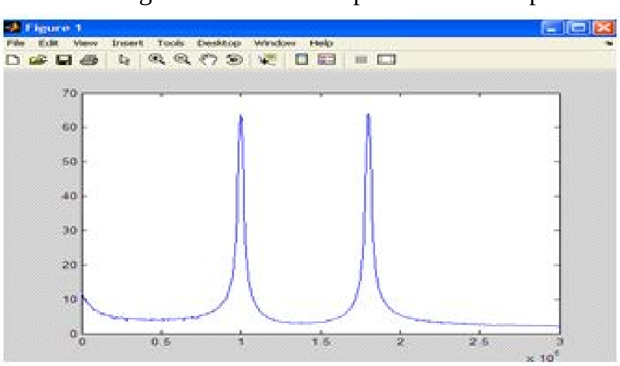


Goertzel algorithm :

Oscillatorfrequeny
minimum=k/N*fs=0.09375*10pow(6)
maximum frequency=0.3*fs+0.09375*fs.

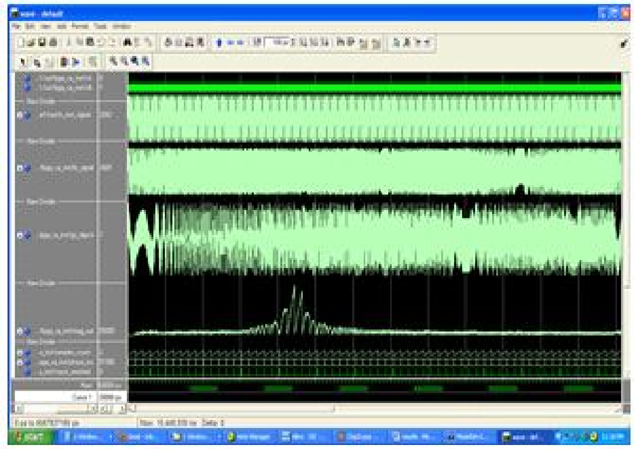
To generating 2 tess signal for testing the spectrum analyzer : test1=100kHz, test2=180kHz, Measurable Range is 0 TO 300 kHz

Goertzel algorithm - mat lab implementation output:

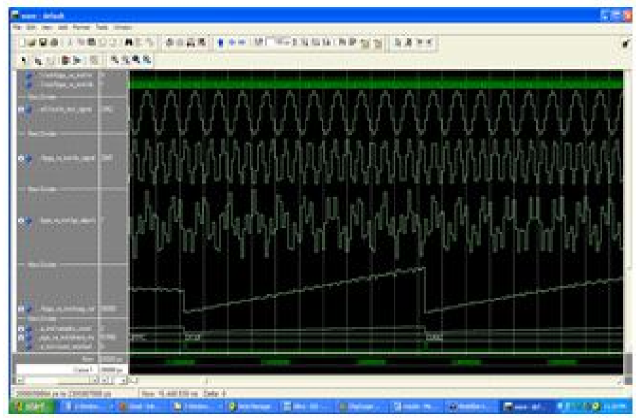


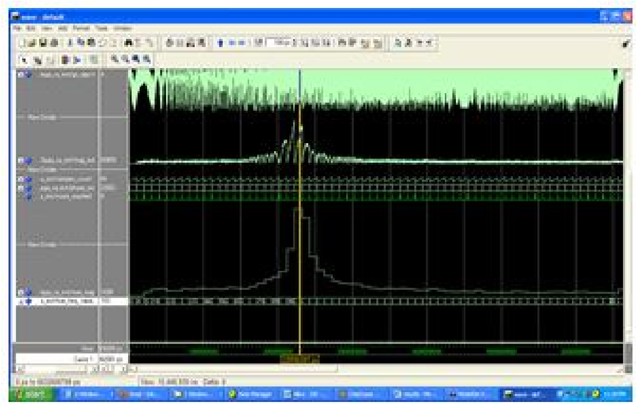
Gortzel based spectrum analyzer

100 KHz and 180 KHz sine waves are given as test signals.



Zoom in over the peak of the spectrum outputs,shown below





VI. Conclusions

We have demonstrated a FPGA Implementation of high area efficient and Low power Goertzel algorithm for spectrum analysis.

Simulation

The VHDL test bench reads the signal samples from file and feeds to the algorithm and outputs are seen from modelsim wave window. It shows that the spectrum estimation is correct by comparing with the MATLAB simulation results.

FPGA verification

The signals with different frequency and characteristics,

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kept in FPGA memory as test inputs. The algorithm output verified with chipscope tool. Area utilization from synthesis report , used to prove the area efficency. (in comparison with the standard area occupancy of FFT core) Power analysis, carried out using Xilinx Xpower tool.

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